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Dynamic behaviour of one-dimensional flow multistream heat exchangers and their networks

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Abstract

The dynamic behaviour of one-dimensional flow (cocurrent and countercurrent) multistream heat exchangers and their networks is modelled and simulated. The problems can be classified into two types: (1) dynamic responses to arbitrary temperature transients and to sudden flow rate transients from a uniform temperature initial condition or a steady-state condition, which yield a linear mathematical model; (2) dynamic responses to disturbances in thermal flow rates, heat transfer coefficients or flow distributions, which are non-linear problems and should be solved numerically. A linearized model is developed to solve the non-linear problems with small disturbances. The linear model and the linearized model for small disturbances are solved by means of Laplace transform and numerical inverse algorithm. Introducing four matching matrices, the general solution can be applied to various types of one-dimensional flow multistream heat exchangers such as shell-and-tube heat exchangers and plate heat exchangers as well as their networks. The time delays in connecting and bypass pipes are included in the models. The software TAIHE (transient analysis in heat exchangers) is further developed to include the present general solution and is applied to the simulation of fluid temperature responses of multistream heat exchangers. Examples are given to illustrate the procedures in detail.

Keywords: Heat exchangers; Systems; Transient

1. Introduction

Multistream heat exchangers are widely used in process industries such as gas processing and petrochemical industries to exchange heat among more than two fluids with different supply temperatures owing to their higher efficiency, more compact structure and lower costs than two stream heat exchanger networks. The use of multistream heat exchangers is more cost-effective and can offer significant advantages over conventional two-stream heat exchangers in certain applications, especially in cryogenic plants [1–3].

The steady-state behaviour of multistream heat exchangers has well been investigated numerically [4–13] and analytically [14–26]. However, in industry heat exchangers and their networks frequently undergo transients resulting from external load variations and regulations. In many industrial processes and operations, such as in nuclear reactors, power plants and chemical processes, precise simulations of the transient responses of heat exchangers are required. Optimal operation, treatment of accidents and real-time control and regulation also demand more accurate description of the time-domain behaviour of heat exchangers. With the computing power available today, modern control theories are also developed to control thermal systems using real-time dynamic models of heat exchangers.

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Nomenclature

F	heat transfer area, m^2
$\mathbf{G}$	interconnection matrix
$\mathbf{G}'$	entrance matching matrix
$\mathbf{G}''$	exit matching matrix
$\mathbf{G}'''$	bypass matrix
$\mathbf{H}$	matrix of eigenvectors of the governing equation system
k	overall heat transfer coefficient, W/m^2 K
L	length of the heat exchanger, m; also Laplace transform operator
M	number of channels; also total mass, kg
M_w	number of walls
N'	number of stream entrances
N''	number of stream exits
s	Laplace parameter, s
$\mathbf{T}$	fluid temperature vector, K
t	temperature, K
U	heat transfer parameter, $U = \alpha F/L$, W/m K
U^*	overall heat transfer parameter, $U^* = kF/L$, W/m K
W	$W = Mc_p/L$, heat capacity per unit length, J/m K
$\dot{W}$	thermal flow rate, W/K
x	spatial coordinate along the flow path of the heat exchanger, m

Greek symbols

α	heat transfer coefficient, W/m^2 K
Λ	vector of eigenvalues of the governing equation system
$\mathbf{\Theta}$	excess fluid temperature vector, $\mathbf{\Theta} = \mathbf{T} - \hat{\mathbf{T}}$, K
θ	excess temperature, $\theta = t - \hat{t}$, K
$\Delta\tau$	time delay in connecting pipe, s
τ	time, s

Superscripts

$'$	entrance
$''$	exit
$\hat{}$	steady state
$\bar{}$	parameters at new operating point
$\tilde{}$	Laplace transform

Subscript

w	wall

Since the first dynamic model presented and solved by Anzelius [27] for the heat transfer between a porous medium and a fluid passing through it, many studies have been made. Historical reviews of earlier investigations on modelling and dynamics of heat exchangers were given by Kays and London [28] and Kanoh [29]. Shah [30] formulated the transient response problems of one-dimensional flow heat exchangers and summarized the available specific solutions for counterflow and cross-flow heat exchangers and thermal regenerators subjected to a step change in inlet temperature and/or flow rate of one or both fluids. New developments in dynamic analysis of heat exchangers were reviewed by Roetzel [31]. Systematic description of the dynamic behaviour of heat exchangers was provided by Roetzel and Xuan [32], in which the linear problems of multipass heat exchangers are solved.

For a long time the investigations of dynamic behaviour of multistream heat exchangers were restricted to the plate-fin type since the most multistream heat exchangers used in industries are of this type. Dynamic behaviour of multistream plate-fin heat exchangers was first studied numerically by Pingaud et al. [33]. In their model the dynamics of fins were simplified by the use of fin efficiency derived from the steady-state analysis, so that the plate-fin heat exchangers can be treated as multichannel heat exchangers. With modern computer technology and numerical methods there is no problem to simulate dynamic responses of multistream heat exchangers. However, the numerical methods have some limitations, mainly numerical errors and time-consuming computation. Therefore, great efforts have been done in order to get more accurate, simple and rapid dynamic simulations. The lumped parameter model was developed by Cai and his coworkers [34–38], which provides the dynamic behaviour of the apparatus without dealing with detailed local temperature distributions. They compared different forms of lumped parameter model with experimental results and found that the model using the outlet temperatures as lumped parameters and the logarithmic mean temperature differences as the driving forces would give the best results [34]. The coefficients appearing in the transfer functions were determined by experiments. Ye [39] further obtained the outlet fluid temperature responses by inversing the transfer functions into the time domain. The coefficients of transfer functions were determined by parameter matching between the analytical outlet temperature variation and the numerical results obtained from a distributed parameter model in which the transient component of the heat flux in fins is neglected [35,38]. Luo and Roetzel [40,41] analytically investigated the effect of the fin dynamics on the temperature responses of plate-fin heat transfer surfaces and found that the effect of lateral heat conduction resistance along the fins should be considered if the fins have higher values of

Biot number (lower fin efficiency), such as stainless steel fins with a high ratio of height to space. Using a numerical method, Luo and Roetzel [42] solved the energy equations of fluids, separating plates and fins simultaneously and compared their model with the traditional model without consideration of fin dynamics. The difference between them is significant for small values of NTU. However, compact heat exchangers usually have large values of NTU and consequently Pingaud's model would be a good approach.

Recently, Roetzel et al. [43] proposed a general model for dynamic responses of one-dimensional flow multistream heat exchangers and their networks. The mathematical model is solved by the use of Laplace transform and numerical inverse algorithm. Realizing the importance of systematic analysis in process design and control, they introduced three connection matrices so that their solution can be directly applied to heat exchanger networks. In the present paper, their method is further developed to include the bypass controls. The software TAIHE (transient analysis in heat exchangers) has been developed to solve the mathematical models using the Laplace transform and numerical inverse algorithm. Examples are given to illustrate the procedures.

2. General mathematical model

Consider a generalized multistream heat exchanger which consists of M fluid channels, M_w solid walls, N' stream entrances and N'' stream exits. N' and N'' can be different due to stream splitting and junction. The fluid flowing through a channel can exchange heat with all solid walls. In the analysis, the following assumptions are made. (1) The mass flow rate and fluid temperature in each channel are considered to be uniform over the cross-section perpendicular to the flow direction (ideal plug-flow). (2) The longitudinal heat conduction in the solid wall is neglected. (3) There is no lateral heat resistances across the walls. (4) There is no heat loss to the environment. (5) The heat transfer coefficients and the properties of the fluids and wall materials are constant. (6) There is no phase change in the exchanger. The governing equation system can be written as

$$W_i \frac{\partial t_i}{\partial \tau} + \dot{W}_i \frac{\partial t_i}{\partial x} = \sum_{j=1}^{M_w} U_{ij}\left(t_{w,j} - t_i\right) \quad (i = 1, \ldots, M), \tag{1}$$

$$W_{w,j} \frac{\partial t_{w,j}}{\partial \tau} = \sum_{i=1}^{M} U_{ij}\left(t_i - t_{w,j}\right) \quad (j = 1, \ldots, M_w), \tag{2}$$

$$\begin{aligned} \tau = 0: \quad & t_i = g_i(x) \quad (i = 1, \ldots, M), \\ & t_{w,j} = g_{w,j}(x) \quad (j = 1, \ldots, M_w), \end{aligned} \tag{3}$$

in which W_i and $W_{w,j}$ are the heat capacities of fluid i and wall j per unit length, $\dot{W}_i$ is the thermal flow rate of fluid i and U_{ij} is the heat transfer parameter $\alpha F/L$ between fluid i and wall j.

To specify the boundary conditions and bypasses in a general form, four matching matrices are introduced and defined as follows.

Interconnection matrix $\mathbf{G}$: It is an $M \times M$ matrix whose elements g_{ij} are defined as the ratio of the thermal flow rate flowing from channel j into channel i to that flowing through channel i.

Entrance matching matrix $\mathbf{G}'$: It is an $M \times N'$ matrix whose elements g'_{ik} are defined as the ratio of the thermal flow rate flowing from the entrance k to channel i to that flowing through channel i.

Exit matching matrix $\mathbf{G}''$: It is an $N'' \times M$ matrix whose elements g''_{li} are defined as the ratio of the thermal flow rate flowing from channel i to the exit l to that flowing out of exit l.

Bypass matrix $\mathbf{G}'''$: It is an $N'' \times N'$ matrix whose elements g'''_{lk} are defined as the ratio of the thermal flow rate flowing from entrance k to exit l to that flowing out of exit l.

The energy balances at the inlets of M channels yield

$$t_i(\tau, x'_i) = \sum_{k=1}^{N'} g'_{ik} t'_k(\tau - \Delta\tau'_{ik}) + \sum_{j=1}^{M} g_{ij} t_j(\tau - \Delta\tau_{ij}, x''_j) \quad (i = 1, \ldots, M), \tag{4}$$

in which x'_i and x''_i are the coordinates of the inlet and outlet of channel i, respectively. t'_k is the supply temperature of stream k. $\Delta\tau'_{ik}$ and $\Delta\tau_{ij}$ are the time delays from entrance k to the inlet of channel i and those from the outlet of channel j to the inlet of channel i, respectively. Similarly, the energy balances at the exits of N'' streams read

$$t''_l(\tau) = \sum_{k=1}^{N'} g'''_{lk} t'_k(\tau - \Delta\tau''_{lk}) + \sum_{i=1}^{M} g''_{li} t_i(\tau - \Delta\tau''_{li}, x''_i) \quad (l = 1, \ldots, N''), \tag{5}$$

where $\Delta\tau''_{li}$ and $\Delta\tau'''_{lk}$ are the time delays from the outlet of channel i to exit l and those from entrance k to exit l, respectively.

2.1. A linear model for inlet temperature disturbances

Consider a heat exchanger which runs at first at a steady state, of which all the parameters and temperatures are denoted with "$\hat{}$". The steady-state temperature distributions $\hat{t}_i$ were obtained by Roetzel and Luo [24] as follows:

$$\widehat{\mathbf{T}} = \widehat{\mathbf{H}} \, e^{\hat{\Lambda} x} \widehat{\mathbf{D}}, \tag{6}$$

in which $\widehat{\mathbf{T}} = [\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_M]^{\mathrm{T}}$ is the vector of steady-state fluid temperature distributions in channels, $e^{\hat{\Lambda} x} = \mathrm{diag}\left\{e^{\hat{\lambda}_i x}\right\}$ is a diagonal matrix and $\hat{\lambda}_i$ $(i = 1, \ldots, M)$ are the eigenvalues of coefficient matrix of the governing equation system, $\hat{\mathbf{A}}$, whose elements read

$$\hat{a}_{ij} = \frac{1}{\widehat{\dot{W}}_i} \sum_{n=1}^{M_w} \widehat{U}_{in} \left(\frac{\widehat{U}_{jn}}{\sum_{m=1}^{M} \widehat{U}_{mn}} - \delta_{ij} \right) \quad (i,j = 1,\ldots,M), \tag{7}$$

or using the overall heat transfer parameter,

$$\widehat{U}_{ij}^* = (kF/L)_{ij} = \frac{1}{\widehat{\dot{W}}_i} \sum_{n=1}^{M_w} \frac{\widehat{U}_{in}\widehat{U}_{jn}}{\sum_{m=1}^{M} \widehat{U}_{mn}} \quad (i,j = 1,\ldots,M), \tag{8}$$

which yields

$$\hat{a}_{ij} = \frac{1}{\widehat{\dot{W}}_i} \left(\widehat{U}_{ij}^* - \delta_{ij} \sum_{k=1}^{M} \widehat{U}_{ik}^* \right) \quad (i,j = 1,\ldots,M). \tag{9}$$

The Kronecker symbol δ_{ij} used here is defined as

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases} \tag{10}$$

In Eq. (6) $\widehat{\mathbf{H}}$ is an $M \times M$ square matrix whose columns are the eigenvectors of the corresponding eigenvalues. $\widehat{\mathbf{V}}'$ and $\widehat{\mathbf{V}}''$ are two $M \times M$ matrices defined as

$$\widehat{\mathbf{V}}' = \left\{ \hat{v}'_{ij} \right\}_{M\times M} = \left\{ \hat{h}_{ij} e^{\hat{\lambda}_j \hat{x}'_i} \right\}_{M\times M}, \tag{11}$$

$$\widehat{\mathbf{V}}'' = \left\{ \hat{v}''_{ij} \right\}_{M\times M} = \left\{ \hat{h}_{ij} e^{\hat{\lambda}_j \hat{x}''_i} \right\}_{M\times M}, \tag{12}$$

respectively. The coefficient vector $\widehat{\mathbf{D}}$ is determined by the supply temperature vector of streams $\widehat{\mathbf{T}}' = [\hat{t}'_1, \hat{t}'_2, \ldots, \hat{t}'_N]^{\mathrm{T}}$ as

$$\widehat{\mathbf{D}} = (\widehat{\mathbf{V}}' - \widehat{\mathbf{G}}\widehat{\mathbf{V}}'')^{-1}\widehat{\mathbf{G}}'\widehat{\mathbf{T}}'. \tag{13}$$

Eq. (6) is valid only if the eigenvalues differ from each other. It has been proved that all eigenvalues of matrix $\mathbf{A}$ are real, however, Eq. (6) might have multiple eigenvalues [19–21]. A practical method to avoid multiple eigenvalues is to add small deviations to the thermal flow rates $\widehat{\dot{W}}_i$. Such small deviations have almost no effect on the results.

The exit temperature vector of streams is obtained according to the energy balance at the exits of streams,

$$\widehat{\mathbf{T}}'' = [\widehat{\mathbf{G}}''' + \widehat{\mathbf{G}}''\widehat{\mathbf{V}}''(\widehat{\mathbf{V}}' - \widehat{\mathbf{G}}\widehat{\mathbf{V}}'')^{-1}\widehat{\mathbf{G}}']\widehat{\mathbf{T}}'. \tag{14}$$

We assume that at $\tau = 0$ the exchanger experiences sudden changes in thermal flow rate vector $\dot{\mathbf{W}}$, heat transfer parameter matrix $\mathbf{U}$, matching matrices $\mathbf{G}$, $\mathbf{G}'$, $\mathbf{G}''$ and $\mathbf{G}'''$ and flow directions. They are kept constant for $\tau > 0$. The parameters at the new operating point are denoted with "-". It is assumed that (1) the time delays in the connecting and bypass pipes are constant and (2) heat capacities of connecting and bypass pipes are negligible. If these two assumptions cannot be satisfied for some connecting and bypass pipes, they can alternatively be treated as additional channels of the exchanger or exchanger network.

Introducing the excess temperatures $\theta_i = t_i - \hat{t}_i$ and $\theta_{w,j} = t_{w,j} - \hat{t}_{w,j}$ and substituting them in to Eqs. (1)–(5) with the assumption that the heat exchanger runs at first at the steady state, the governing equation system reads

$$W_i \frac{\partial \theta_i}{\partial \tau} + \overline{\dot{W}}_i \frac{\partial \theta_i}{\partial x} + \sum_{j=1}^{M_w} \overline{U}_{ij}(\theta_i - \theta_{w,j}) = \sum_{j=1}^{M_w} \left(\overline{U}_{ij} - \frac{\overline{\dot{W}}_i}{\widehat{\dot{W}}_i} \widehat{U}_{ij} \right) (\hat{t}_{w,j} - \hat{t}_i) \quad (i = 1,\ldots,M), \tag{15}$$

$$W_{w,j} \frac{\partial \theta_{w,j}}{\partial \tau} + \sum_{i=1}^{M} \overline{U}_{ij}(\theta_{w,j} - \theta_i) = \sum_{i=1}^{M} \overline{U}_{ij}\left(\hat{t}_i - \hat{t}_{w,j}\right) \quad (j = 1,\ldots,M_w), \tag{16}$$

$$\tau = 0: \quad \theta_i = 0 \quad (i = 1,\ldots,M), \quad \theta_{w,j} = 0 \quad (j = 1,\ldots,M_w), \tag{17}$$

$$\theta_i(\tau, x'_i) - \sum_{k=1}^{N'} \overline{g}'_{ik}\theta'_k(\tau - \Delta\tau'_{ik}) - \sum_{j=1}^{M} \overline{g}_{ij}\theta_j\left(\tau - \Delta\tau_{ij}, x''_j\right) = -\hat{t}_i(x'_i) + \sum_{k=1}^{N'} \overline{g}'_{ik}\hat{t}'_k + \sum_{j=1}^{M} \overline{g}_{ij}\hat{t}_j(x''_j) \quad (i = 1,\ldots,M), \tag{18}$$

$$\theta''_l(\tau) - \sum_{k=1}^{N'} \overline{g}'''_{lk}\theta'_k(\tau - \Delta\tau'''_{lk}) - \sum_{i=1}^{M} \overline{g}''_{li}\theta_i(\tau - \Delta\tau''_{li}, x''_i) = -\hat{t}''_l + \sum_{k=1}^{N'} \overline{g}'''_{lk}\hat{t}'_k + \sum_{i=1}^{M} \overline{g}''_{li}\hat{t}_i(x''_i) \quad (l = 1,\ldots,N''). \tag{19}$$

2.2. A linearized model for flow disturbances

Now we consider a heat exchanger which runs at first at a steady state, then the exchanger experiences flow and inlet temperature disturbances around its new operating point. The former will result in disturbances in thermal flow rate vector $\dot{\mathbf{W}}$, heat transfer parameter matrix $\mathbf{U}$ and matching matrices $\mathbf{G}$, $\mathbf{G}'$, $\mathbf{G}''$ and $\mathbf{G}'''$. The governing equation system for flow disturbances is non-linear. Assume that the disturbances are relative small so that the products of the disturbances and excess temperatures are negligible. Therefore the parameters appearing in the non-linear term can be replaced with their mean values at the new operating point denoted with "-", which yields the following linearized model:

$$W_i \frac{\partial \theta_i}{\partial \tau} + \overline{\dot{W}}_i \frac{\partial \theta_i}{\partial x} + \sum_{j=1}^{M_w} \overline{U}_{ij}(\theta_i - \theta_{w,j}) = \sum_{j=1}^{M_w} \left(U_{ij} - \frac{\dot{W}_i}{\widehat{\dot{W}}_i} \widehat{U}_{ij} \right) \left(\hat{t}_{w,j} - \hat{t}_i \right) \quad (i = 1, \dots, M), \tag{20}$$

$$W_{w,j} \frac{\partial \theta_{w,j}}{\partial \tau} + \sum_{i=1}^{M} \overline{U}_{ij}(\theta_{w,j} - \theta_i) = \sum_{i=1}^{M} U_{ij} \left(\hat{t}_i - \hat{t}_{w,j} \right) \quad (j = 1, \dots, M_w), \tag{21}$$

$$\tau = 0: \quad \theta_i = 0 \quad (i = 1, \dots, M), \quad \theta_{w,j} = 0 \quad (j = 1, \dots, M_w), \tag{22}$$

$$\theta_i(\tau, x_i') - \sum_{k=1}^{N'} \overline{g}'_{ik} \theta'_k(\tau - \Delta\tau'_{ik}) - \sum_{j=1}^{M} \overline{g}_{ij} \theta_j\left(\tau - \Delta\tau_{ij}, x_j''\right) = -\hat{t}_i(x_i') + \sum_{k=1}^{N'} g'_{ik} \hat{t}'_k + \sum_{j=1}^{M} g_{ij} \hat{t}_j\left(x_j''\right) \quad (i = 1, \dots, M), \tag{23}$$

$$\theta_l''(\tau) - \sum_{k=1}^{N'} \overline{g}'''_{lk} \theta'_k(\tau - \Delta\tau'''_{lk}) - \sum_{i=1}^{M} \overline{g}''_{li} \theta_i(\tau - \Delta\tau''_{li}, x_i'') = -\hat{t}_l'' + \sum_{k=1}^{N'} g'''_{lk} \hat{t}'_k + \sum_{i=1}^{M} g''_{li} \hat{t}_i(x_i'') \quad (l = 1, \dots, N''). \tag{24}$$

The governing equation system, Eqs. (20)–(24), is an extended form of Eqs. (15)–(19), which takes the further flow variations with time around the new operating point into account. However, for non-linear problems, i.e., $\dot{\mathbf{W}}$, $\mathbf{U}$, $\mathbf{G}$, $\mathbf{G}'$, $\mathbf{G}''$ and $\mathbf{G}'''$ are functions of time for $\tau > 0$, therefore, the linearized model is an approximate one which is only valid for small disturbances in inlet fluid temperatures and mass flow rates.

3. The analytical solution with numerical inverse algorithm

The governing equation systems for linear problems and linearized non-linear problems are solved by means of Laplace transform. Applying the Laplace transform to Eqs. (20)–(24), we have,

$$\frac{d\widetilde{\mathbf{\Theta}}}{dx} = \mathbf{A}\widetilde{\mathbf{\Theta}} + \mathbf{B}\widehat{\mathbf{T}}, \tag{25}$$

in which $\widetilde{\mathbf{\Theta}} = [\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_M]^{\mathrm{T}}$. The elements of $M \times M$ matrices $\mathbf{A}$ and $\mathbf{B}$ in Eq. (25) are given as

$$a_{ij} = \frac{1}{\overline{\dot{W}}_i} \left[\sum_{n=1}^{M_w} \frac{\overline{U}_{in}\overline{U}_{jn}}{sW_{w,n} + \sum_{m=1}^{M} \overline{U}_{mn}} - \delta_{ij}\left(sW_i + \sum_{n=1}^{M_w} \overline{U}_{in} \right) \right] \quad (i, j = 1, \dots, M), \tag{26}$$

$$b_{ij} = \frac{\widetilde{\dot{W}}_i}{\overline{\dot{W}}_i \widehat{\dot{W}}_i} \sum_{n=1}^{M_w} \widehat{U}_{in} \left(\delta_{ij} - \frac{\widehat{U}_{jn}}{\sum_{m=1}^{M} \widehat{U}_{mn}} \right) - \frac{1}{\overline{\dot{W}}_i} \sum_{k=1}^{M} \sum_{n=1}^{M_w} \left(\delta_{ik} - \frac{\overline{U}_{in}}{sW_{w,n} + \sum_{m=1}^{M} \overline{U}_{mn}} \right) \times \left(\delta_{jk} - \frac{\widehat{U}_{jn}}{\sum_{m=1}^{M} \widehat{U}_{mn}} \right) \widetilde{U}_{kn} \quad (i, j = 1, \dots, M), \tag{27}$$

in which $\widetilde{\dot{W}}$ and $\widetilde{U}$ are the Laplace transforms of thermal flow rates and heat transfer parameters, respectively, which can be written as

$$\widetilde{\dot{W}}(s) = L[\Delta\dot{W}(\tau) + \overline{\dot{W}}] = \Delta\widetilde{\dot{W}}(s) + \overline{\dot{W}}/s, \tag{28}$$

$$\widetilde{U}(s) = L[\Delta U(\tau) + \overline{U}] = \Delta\widetilde{U}(s) + \overline{U}/s. \tag{29}$$

The boundary conditions in the Laplace plane can be expressed in the matrix form as

$$\widetilde{\mathbf{\Theta}}(\mathbf{x}') = \widetilde{\overline{\mathbf{G}}}{}'\widetilde{\mathbf{\Theta}}' + \widetilde{\overline{\mathbf{G}}}\widetilde{\mathbf{\Theta}}(\mathbf{x}'') - \frac{1}{s}\widehat{\mathbf{T}}(\mathbf{x}') + \widetilde{\mathbf{G}}'\widehat{\mathbf{T}}' + \widetilde{\mathbf{G}}\widehat{\mathbf{T}}(\mathbf{x}''), \tag{30}$$

$$\widetilde{\mathbf{\Theta}}'' = \widetilde{\overline{\mathbf{G}}}{}'''\widetilde{\mathbf{\Theta}}' + \widetilde{\overline{\mathbf{G}}}{}''\widetilde{\mathbf{\Theta}}(\mathbf{x}'') - \frac{1}{s}\widehat{\mathbf{T}}'' + \widetilde{\mathbf{G}}'''\widehat{\mathbf{T}}' + \widetilde{\mathbf{G}}''\widehat{\mathbf{T}}(\mathbf{x}''), \tag{31}$$

in which $\widetilde{\overline{\mathbf{G}}}$, $\widetilde{\overline{\mathbf{G}}}{}'$, $\widetilde{\overline{\mathbf{G}}}{}''$ and $\widetilde{\overline{\mathbf{G}}}{}'''$ are matching matrices with time delay, whose elements are given by $\overline{g}_{ij}\,\mathrm{e}^{-\Delta\tau_{ij}s}$, $\overline{g}'_{ik}\,\mathrm{e}^{-\Delta\tau'_{ik}s}$, $\overline{g}''_{li}\,\mathrm{e}^{-\Delta\tau''_{li}s}$ and $\overline{g}'''_{lk}\,\mathrm{e}^{-\Delta\tau'''_{lk}s}$, respectively.

According to the theory of linear algebra, the general solution of Eq. (25) is obtained by substituting the steady-state temperature distribution, Eq. (6), into Eq. (25) and integrating the inhomogeneous term, which yields

$$\widetilde{\mathbf{\Theta}} = \mathbf{H}\,\mathrm{e}^{\Lambda x}\mathbf{D} + \mathbf{C}\,\mathrm{e}^{\hat{\Lambda} x}\widehat{\mathbf{D}}, \tag{32}$$

where $\Lambda = \{\lambda_i\}_{M\times 1}$ contains the eigenvalues of $\mathbf{A}$ and $\mathbf{H} = \{h_{ij}\}_{M\times M}$ contains the eigenvectors of $\mathbf{A}$. The term $\mathbf{C}\,\mathrm{e}^{\hat{\Lambda} x}\widehat{\mathbf{D}}$ is the special solution of the inhomogeneous ordinary differential equation system, Eq. (25), in which

$$\mathbf{C} = \{c_{ij}\}_{M\times M} = \left\{ \sum_{m=1}^{M} \frac{h_{im}c'_{mj}}{\hat{\lambda}_j - \lambda_m} \right\}_{M\times M}, \tag{33}$$

$$\mathbf{C}' = \left\{c'_{ij}\right\}_{M\times M} = \mathbf{H}^{-1}\mathbf{B}\widehat{\mathbf{H}}. \tag{34}$$

The coefficient vector $\mathbf{D}$ in Eq. (32) is determined by substitution of Eqs. (6) and (32) into Eq. (30), yielding

$$\mathbf{D} = (\mathbf{V}' - \tilde{\tilde{\mathbf{G}}}\mathbf{V}'')^{-1}\left[\tilde{\tilde{\mathbf{G}}}'\tilde{\mathbf{\Theta}}' + \tilde{\mathbf{G}}'\hat{\mathbf{T}}' - \left(\overline{\mathbf{V}}' - \tilde{\tilde{\mathbf{G}}}\overline{\mathbf{V}}'' + \frac{1}{s}\hat{\overline{\mathbf{V}}}' - \tilde{\mathbf{G}}\hat{\overline{\mathbf{V}}}''\right)\hat{\mathbf{D}}\right], \tag{35}$$

in which $\hat{\mathbf{D}}$ is calculated with Eq. (13),

$$\mathbf{V}' = \left\{v'_{ij}\right\}_{M\times M} = \left\{h_{ij}\,\mathrm{e}^{\lambda_j x'_i}\right\}_{M\times M}, \tag{36}$$

$$\mathbf{V}'' = \left\{v''_{ij}\right\}_{M\times M} = \left\{h_{ij}\,\mathrm{e}^{\lambda_j x''_i}\right\}_{M\times M}, \tag{37}$$

$$\overline{\mathbf{V}}' = \left\{\bar{v}'_{ij}\right\}_{M\times M} = \left\{c_{ij}\,\mathrm{e}^{\lambda_j x'_i}\right\}_{M\times M}, \tag{38}$$

$$\overline{\mathbf{V}}'' = \left\{\bar{v}''_{ij}\right\}_{M\times M} = \left\{c_{ij}\,\mathrm{e}^{\lambda_j x''_i}\right\}_{M\times M}, \tag{39}$$

$$\hat{\overline{\mathbf{V}}}' = \left\{\hat{\bar{v}}'_{ij}\right\}_{M\times M} = \left\{\hat{h}_{ij}\,\mathrm{e}^{\lambda_j x'_i}\right\}_{M\times M}, \tag{40}$$

$$\hat{\overline{\mathbf{V}}}'' = \left\{\hat{\bar{v}}''_{ij}\right\}_{M\times M} = \left\{\hat{h}_{ij}\,\mathrm{e}^{\lambda_j x''_i}\right\}_{M\times M}. \tag{41}$$

The exit temperature vector of streams is obtained from Eq. (31), which can be written as

$$\tilde{\mathbf{\Theta}}'' = \tilde{\tilde{\mathbf{G}}}'''\tilde{\mathbf{\Theta}}' + \tilde{\tilde{\mathbf{G}}}''\mathbf{V}''\mathbf{D} + \left(\tilde{\mathbf{G}}''' - \frac{1}{s}\hat{\mathbf{G}}'''\right)\hat{\mathbf{T}}' + \left(\tilde{\mathbf{G}}''\hat{\overline{\mathbf{V}}}'' + \tilde{\tilde{\mathbf{G}}}''\overline{\mathbf{V}}'' - \frac{1}{s}\hat{\mathbf{G}}''\hat{\mathbf{V}}''\right)\hat{\mathbf{D}}. \tag{42}$$

If the deviations from a steady state occur only in inlet stream temperatures, Eq. (42) reduces to

$$\tilde{\mathbf{\Theta}}'' = \left[\tilde{\mathbf{G}}''' + \tilde{\mathbf{G}}''\mathbf{V}''(\mathbf{V}' - \tilde{\mathbf{G}}\mathbf{V}'')^{-1}\tilde{\mathbf{G}}'\right]\tilde{\mathbf{\Theta}}'. \tag{43}$$

After having the analytical solution in the Laplace plane, the temperature responses in the real-time domain can be obtained by means of the numerical inversion using the fast Fourier transform (FFT) technique [31,44,45],

$$f(z_n) = \frac{\exp(az_n)}{z}\left[\mathrm{Re}\tilde{f}_n - \frac{1}{2}\tilde{f}(a)\right], \tag{44}$$

where $z_n = 2nz/M$, $4/z < a < 5/z$ and the function

$$\tilde{f}_n = \sum_{k=0}^{M-1}\tilde{f}\left(a + \frac{\mathrm{i}k\pi}{z}\right)\exp\left(\mathrm{i}\frac{2\pi nk}{M}\right) \tag{45}$$

is calculated by means of FFT algorithm.

4. Applications of the general solutions

4.1. Example 1

This example is given by Roetzel and Xuan [46]. It is a 1–3 shell-and-tube heat exchanger. The entrances of the two fluids are located at the opposite ends of the exchanger, as shown in Fig. 1. At first the heat exchanger

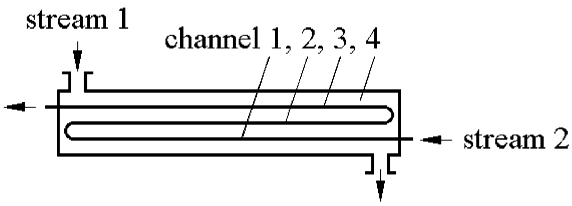

Fig. 1. A shell-and-tube heat exchanger with one shellside pass and three tubeside passes.

runs at a steady state and therefore has a zero excess temperature distribution. Then the inlet excess temperature of the shellside fluid undergoes a sinusoidal change $\theta'_1(\tau) = \sin\tau$. The authors pointed out that the Gaver–Stehfest algorithm [47,48] cannot give correct responses for this problem since the solutions contain oscillatory components. In fact, as has been pointed out by Luo [49], the Gaver–Stehfest algorithm is valid only if the solution in real-time domain is continuous and monotone for $\tau > 0$. Here the problem is resolved with the present algorithm. As shown in Fig. 1, the heat exchanger has four channels, two stream inlets and two stream outlets. The corresponding connecting matrices and the locations of inlets and outlets of channels are given as

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{G}' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{G}'' = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{G}''' = \mathbf{0},$$

$$\mathbf{x}' = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{x}'' = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

respectively, in which the exchanger length is taken as $L = 1$ m. The outlet responses are shown in Fig. 2, together with other parameters used in the calculation.

4.2. Example 2

Consider a heat shifting system consisting of two counterflow heat exchangers coupled by a circulating flow stream, which is used to indirectly transfer heat from a hot stream (stream 1) to a cold stream (stream 2), as shown in Fig. 3. This example is given by Na Ranong [50] and Na Ranong and Roetzel [51]. In the analysis the heat capacities of the shells and heat losses to the environment are neglected. However, the heat capacities of the connecting pipes for the circulating stream are taken into account. The system operates initially at a steady state. The steady-state parameters and connecting matrices are given as follows:

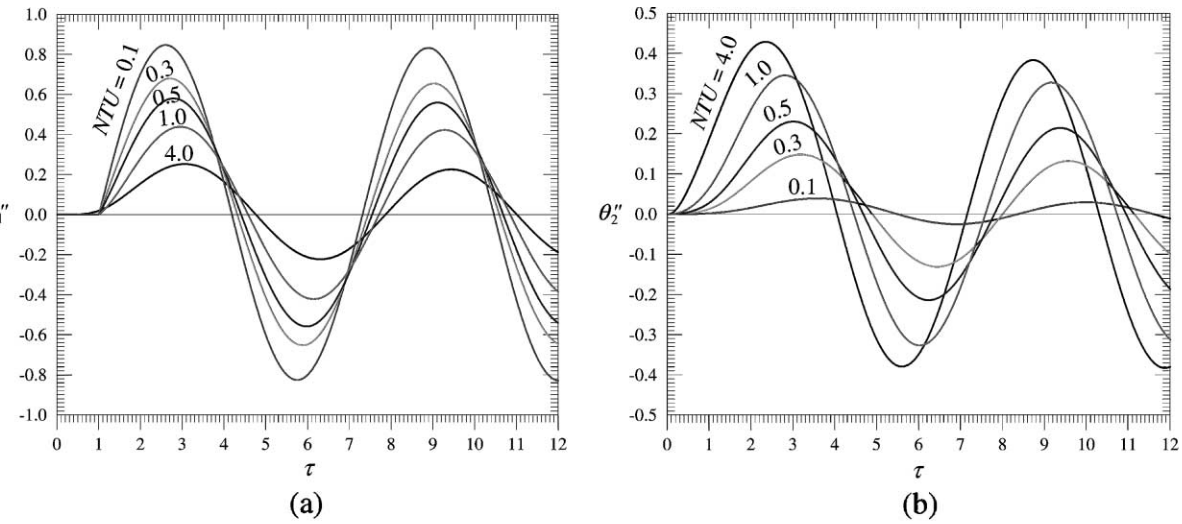

Fig. 2. Exit responses to a shellside sinusoidal inlet temperature change of 1–3 heat exchanger with stream entrances at opposite ends ($|\dot{W}_1| = |\dot{W}_2| = |\dot{W}_3| = |\dot{W}_4| = \dot{W}$, $W_1/\dot{W} = 0.38$, $W_2/\dot{W} = 0.25$, $W_3/\dot{W} = 0.37$, $W_{w1}/\dot{W} = W_{w3}/\dot{W} = 0.35$, $W_{w2}/\dot{W} = 0.30$, $U_{11}/\dot{W} = 0.8\mathrm{NTU}$, $U_{11}/\dot{W} = U_{41}/\dot{W} = U_{33}/\dot{W} = U_{43}/\dot{W} = 0.8\mathrm{NTU}$, $U_{22}/\dot{W} = U_{42}/\dot{W} = 0.4\mathrm{NTU}$, $\theta_1'(\tau) = \sin\tau$, $\theta_2'(\tau) = 0$). (a) shellside fluid (b) tubeside fluid.

$U_{11} = U_{22} = 2,\quad U_{31} = U_{42} = 12,\quad U_{53} = U_{64} = 1,$

other values of U are zero;

$|\dot{W}_1| = |\dot{W}_2| = 0.25,\quad |\dot{W}_3| = |\dot{W}_4| = |\dot{W}_5| = |\dot{W}_6| = 1;$

$W_1 = W_2 = 0,\quad W_3 = W_4 = 1,\quad W_5 = W_6 = 10;$

$W_{w1} = W_{w2} = 1.25,\quad W_{w3} = W_{w4} = 1;$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},\quad \mathbf{G}' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{G}'' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},\quad \mathbf{G}''' = \mathbf{0},$$

$$\mathbf{x}' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},\quad \mathbf{x}'' = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

The system responses to a unit step disturbance in the inlet temperature of stream 1 are shown in Fig. 4. The results are the same as those given by Na Ranong [50]. Since the problem is linear, an easy way to obtain the system responses to arbitrary inlet temperature disturbances is the use of Duhamel's theorem [52],

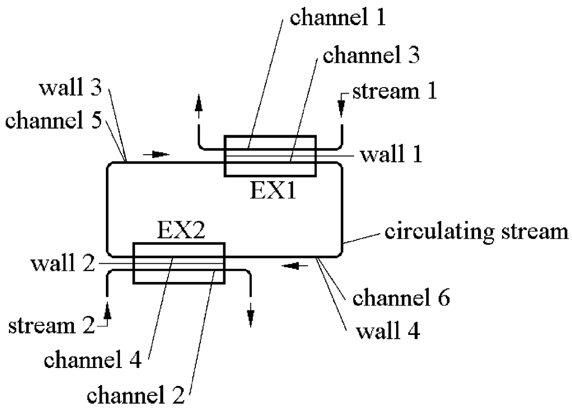

Fig. 3. System of two counterflow heat exchangers coupled by a circulating stream.

$$\theta_1''(\tau) = \int_0^\tau \theta_1''^{U1}(z)\theta_1'^{(1)}(\tau - z)\,\mathrm{d}z + \theta_1''^{U1}(\tau)\theta_1'(0) + \int_0^\tau \theta_1''^{U2}(z)\theta_2'^{(1)}(\tau - z)\,\mathrm{d}z + \theta_1''^{U2}(\tau)\theta_2'(0), \quad (46)$$

$$\theta_2''(\tau) = \int_0^\tau \theta_2''^{U1}(z)\theta_1'^{(1)}(\tau - z)\,\mathrm{d}z + \theta_2''^{U1}(\tau)\theta_1'(0) + \int_0^\tau \theta_2''^{U2}(z)\theta_2'^{(1)}(\tau - z)\,\mathrm{d}z + \theta_2''^{U2}(\tau)\theta_2'(0), \quad (47)$$

in which $\theta_i''^{Uj}$ is the response of the outlet temperature of stream i to a unit step change in the inlet temperature of stream j and $\theta_j'^{(1)}(\tau) = \mathrm{d}\theta_j'(\tau)/\mathrm{d}\tau$ $(i, j = 1, 2)$.

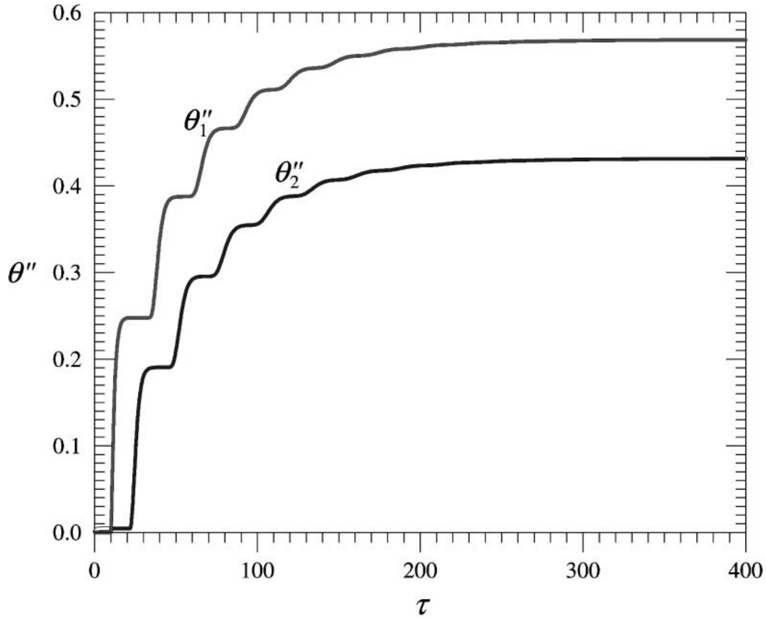

Fig. 4. System responses to a unit step disturbance in the inlet temperature of stream 1 ($\theta'_1(\tau) = 1$).

If the mass flow rates are disturbed, the heat transfer coefficients will also vary with time. By assuming that the properties of the fluids are constant and using the empirical correlation for the Nusselt number,

$$Nu = C\,Re^m\,Pr^n, \tag{48}$$

the heat transfer parameter disturbances can be approximately expressed as

$$U(\tau) = [1 + \sigma(\tau)]^{m-1}\widehat{U}, \tag{49}$$

in which

$$\sigma(\tau) = \frac{\dot{m}(\tau) - \hat{\dot{m}}}{\hat{\dot{m}}}. \tag{50}$$

For a step change in one of the mass flow rates at $\tau = 0$, σ is a constant. Thus we have $\overline{\dot{W}} = (1+\sigma)\widehat{\dot{W}}$, $\overline{U} = (1+\sigma)^{m-1}\widehat{U}$, $\Delta\dot{W}(\tau) = 0$ and $\Delta U(\tau) = 0$. In this case the problem is linear. The system responses of the outlet stream temperatures to step disturbances in mass flow rates are shown in Fig. 5 with solid lines. In order to

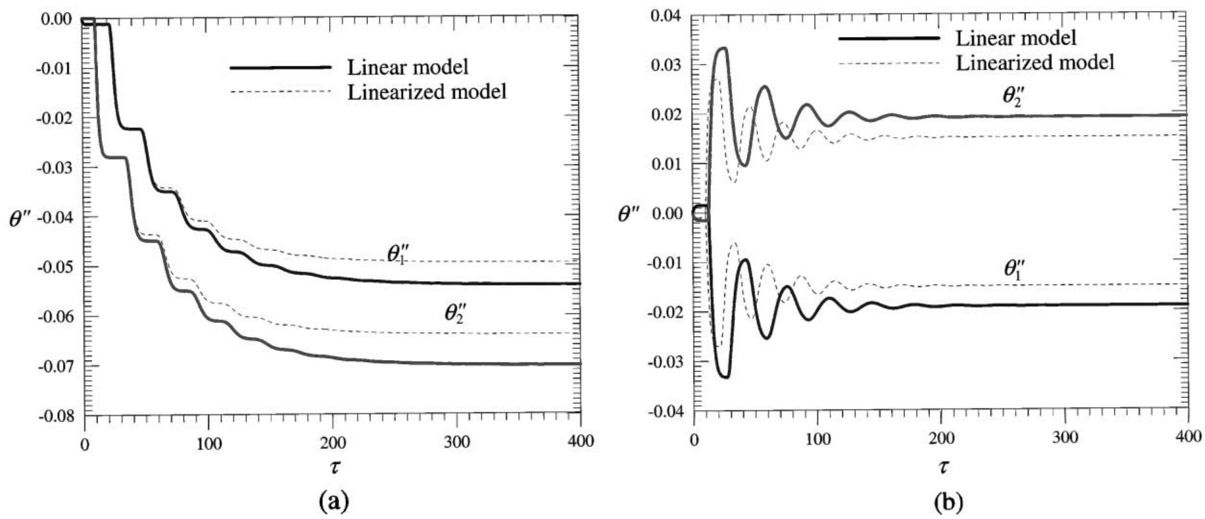

Fig. 5. System responses from a steady state ($\hat{\theta}'_1 = 1$, $\hat{\theta}'_2 = 0$) to step disturbances in the mass flow rates of (a) stream 1 and (b) the circulating stream. (a) $\sigma_1 = -0.2$ (b) $\sigma_i = -0.2$ ($i = 3, 4, 5, 6$).

check the validity of the linearized model, the problem is resolved by setting $\Delta\dot{W}(\tau) = \sigma\widehat{\dot{W}}$, $\Delta U(\tau) = [(1+\sigma)^{m-1} - 1]\widehat{U}$, $\overline{\dot{W}} = \widehat{\dot{W}}$ and $\overline{U} = \widehat{U}$, which yields a non-linear problem. The system responses obtained with the linearized model are shown in Fig. 5 with dotted lines. The deviations between the linear model and linearized model are less than 0.7% of the maximum temperature difference. This error is caused due to omitting the non-linear terms $\Delta\dot{W}(\tau)\theta(\tau)$ and $\Delta U(\tau)\theta(\tau)$. For small disturbances in $\Delta\dot{W}$ and ΔU, the linearized model is a good approach.

5. Conclusions

A general model for predicting the dynamic behaviour of one-dimensional flow (cocurrent and countercurrent) multistream heat exchangers and their networks is proposed. By introducing four connection matrices the solution is general and can directly be applied to one-dimensional flow heat exchanger networks including stream splitting and bypass controls.

The temperature dynamics of heat exchangers can be classified into linear and non-linear problems. Dynamic responses to arbitrary temperature transients, to sudden flow rate transients and to sudden constructional changes (i.e. changes in flow directions, connections, splitting and bypass) from a uniform initial condition or a steady-state condition will yield a linear mathematical model. However, dynamic responses to time-dependent disturbances in thermal flow rates, heat transfer coefficients or flow distributions are non-linear problems and should be solved numerically or approximately by means of linearization. A new linearized model is developed to solve the non-linear problems with small non-linear disturbances. In this model, the disturbances are treated according to the independent parameters appearing in the governing equation system rather than mass flow rates. Since these parameters can always be expressed as functions of mass flow rates, if the mass flow rates change with time, the variations of these parameters with time are also known. With this treatment, a further linearization for these parameters is avoided. The present linearization model is based on the new average operating parameters for $\tau \to \infty$ so that it is possible to predict the temperature dynamics even from a non-steady state because the influence of the initial temperature distributions will vanish after a long time.

A software TAIHE is developed to include the present general solution and applied to the present calculations of system responses to inlet temperature changes and mass flow disturbances. Examples are given in detail for a multipass shell-and-tube heat exchanger and a heat shifting system consisting of two counterflow heat exchangers coupled through a circulating stream. In the latter example the dynamics of the pipe lines are taken into account by considering the pipe lines as flow channels. The software TAIHE is originally part of the selected computer programs developed by Luo and Roetzel in connection with the book *Dynamic Behaviour of Heat Exchangers* [32]. Using TAIHE one can calculate outlet temperature responses to arbitrary inlet temperature changes in multipass shell-and-tube heat exchangers, cross-flow heat exchangers, multistream plate-fin heat exchangers and their networks or arbitrary mass flow disturbances in one-dimensional flow heat exchangers and their networks. Maldistribution is taken into account with the parabolic dispersion model. An analytical procedure is used for rapid calculations of linear and linearized problems. A numerical algorithm with moving grid technique is offered for more accurate calculation of non-linear problems. However, TAIHE has not been published yet. More information about TAIHE is available by contact with the corresponding author.

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